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REDUCTION OF LINE CURRENT HARMONICS IN 3-PHASE OFF-LINE RECTIFIER SYSTEMS  
BY USE OF AN EFFICIENT 3-PHASE TO 9-PHASE AUTOTRANSFORMER CONVERSION TECHNIQUE

DRAFT



12 April 1979

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NOTICE

Because of the degree of current interest in the subject of reducing power line harmonic currents, this report is being released to a limited distribution in ~~██████████~~ DRAFT form. It is expected that editorial and technical modifications will occur prior to issue of the complete report. Additional effort will also be expended in verifying the analysis. It is expected that laboratory testing will be performed in the near future to obtain experimental verification of the analysis. Comments are solicited.

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# REDUCTION OF LINE CURRENT HARMONICS IN 3-PHASE OFF-LINE RECTIFIER SYSTEMS BY USE OF AN EFFICIENT 3-PHASE TO 9-PHASE AUTOTRANSFORMER CONVERSION TECHNIQUE

## SUMMARY

A technique is described which provides a substantial reduction (50 dB or more) of the low frequency harmonics of a 3-phase off-line rectifier system. In a conventional off-line rectifier system, the line <sup>approx</sup> current waveform has a total harmonic distortion (THD) of 78 percent. With the technique described herein, the line current is modified to a nearly sinusoidal waveshape having less than 5 percent THD, predominantly 17th and 19th harmonic. The technique employs autotransformers to achieve a high degree of volumetric and energy efficiency at a substantially lower cost, and with reduced acoustic hum and stray magnetic fields as compared to conventional transformer configurations. A power density of approximately 12 Watts/in<sup>3</sup> or better is expected to be achieved with this technique for applications requiring several kilowatts of input power, when the 20 degree system is employed. For the 40 degree system, a power density of approximately 6 Watts/in<sup>3</sup> or better is expected.

This technique also has the advantage of reducing the line current harmonics without the use of reactive filter elements. It is therefore inherently insensitive to the effects of the negative input impedance characteristics of switching regulator power supplies, which can result in instability and/or a high degree of susceptibility to conducted interference.

## INTRODUCTION

Harmonic line currents produced by rectifier current pulses are a major source of conducted electromagnetic interference (EMI). This form of interference can result in waveform distortion of the power source and degradation of equipment which is sensitive to the frequency spectrum of the interference. In 3-phase systems, the most difficult interference to eliminate tends to be the 5th and 7th harmonics. Although line filters may be employed to reduce the harmonic currents, the low frequencies involved tend to cause the filters to be prohibitively large. Because of considerable

experience with degradation due to such line current harmonics [1]. military and industrial specifications are becoming more and more restrictive with regard to the allowed levels of line current harmonics. MIL-E-16400G, Amendment 1 requires that, for 60 Hz units of 1000 VA or more, no single harmonic from the 2nd to the 32nd may be greater than 3 percent of the fundamental. From the 32nd harmonic to 20kHz, the individual harmonic currents must not be greater than  $100/n$  percent of the fundamental, where  $n$  is the harmonic number. Transformer methods offer a solution which is attractive from the viewpoint of retrofitting existing existing systems to meet more stringent requirements. Investigations are currently underway for such applications [2]. The technique described herein has been shown by analysis to meet the 3 percent and  $100/n$  harmonic current requirement of MIL-E-16400G Amendment 1, except at the 17th harmonic where the line current harmonic is 3.9 percent. A minor degree of filtering would be sufficient to achieve full compliance to these requirements.

## DISCUSSION

When a conventional full-wave bridge capacitor-input rectifier is operated from a 3-phase line, as is quite common in off-line switch-mode power supply applications, the unfiltered line current has a doublet pulse characteristic as shown in Figure 1A. The frequency spectrum of this current waveform shown in Figure 1B indicates that the 5th and 7th harmonics are only several dB below the magnitude of the fundamental frequency component. The 11th and 13th harmonics are also rather large. The Total Harmonic Distortion

of this waveform is 78 percent. Transformers may be employed to convert the 3-phase rectifier system to a Many-phase system. As more phases are added, the line current waveform improves and approaches a pure sinusoid as the number of phases becomes very large.<sup>[2]</sup> Although this approach is very effective, a large penalty must be paid in terms of space, weight, and cost for the use of such transformers. Acoustic hum produced by the transformers may also be objectionable in some applications. *and stray magnetic fields*

The technique presented here provides a very large improvement in the space, weight, and cost factors by making use of the very high efficiency of autotransformers. As shown in Figure 2, the volt-ampere capacity requirement of an autotransformer is considerably less than that of a conventional transformer when the input/output voltage transformation ratio is close to unity. As the input/output voltage transformation ratio approaches unity, the volt-ampere capacity requirement of the autotransformer approaches zero. Therefore, in applications in which the <sup>input/output</sup> voltage transformation ratio is close to unity, and where DC isolation is not required, the use of autotransformers instead of conventional transformers results in a large reduction of size, weight, and cost. Acoustic hum should also be substantially reduced with the smaller transformers. *and stray magnetic fields*

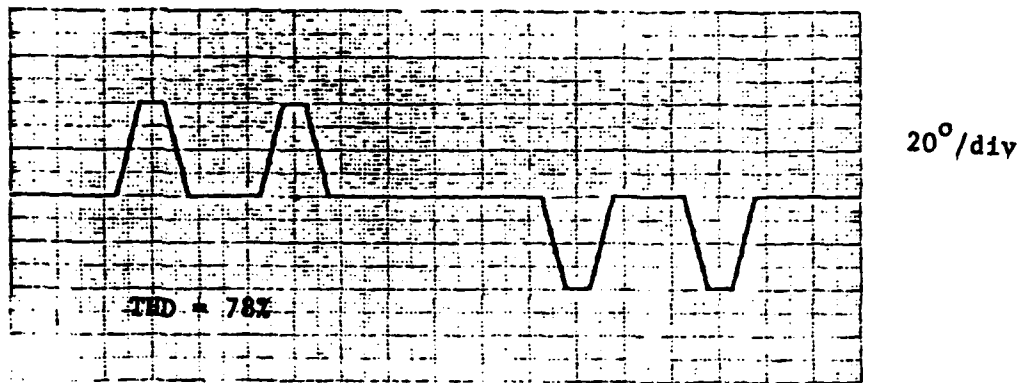


Figure 1A Typical line current waveform for conventional 3-phase full-wave bridge rectifier with capacitor input filter

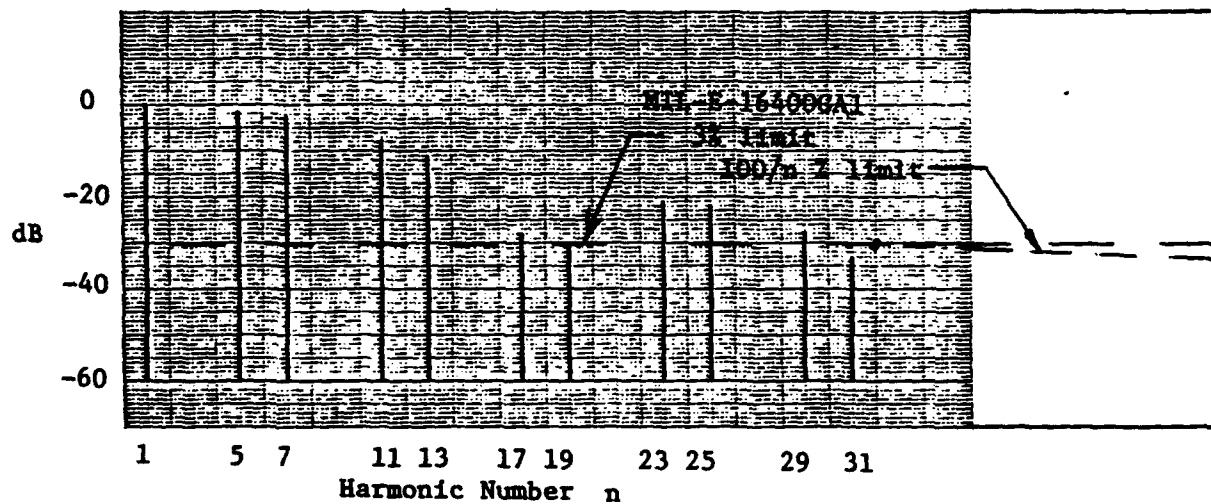
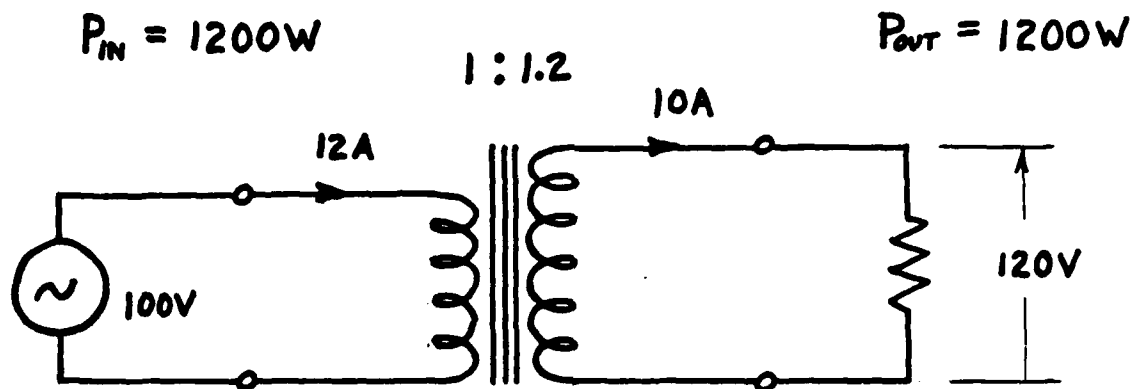
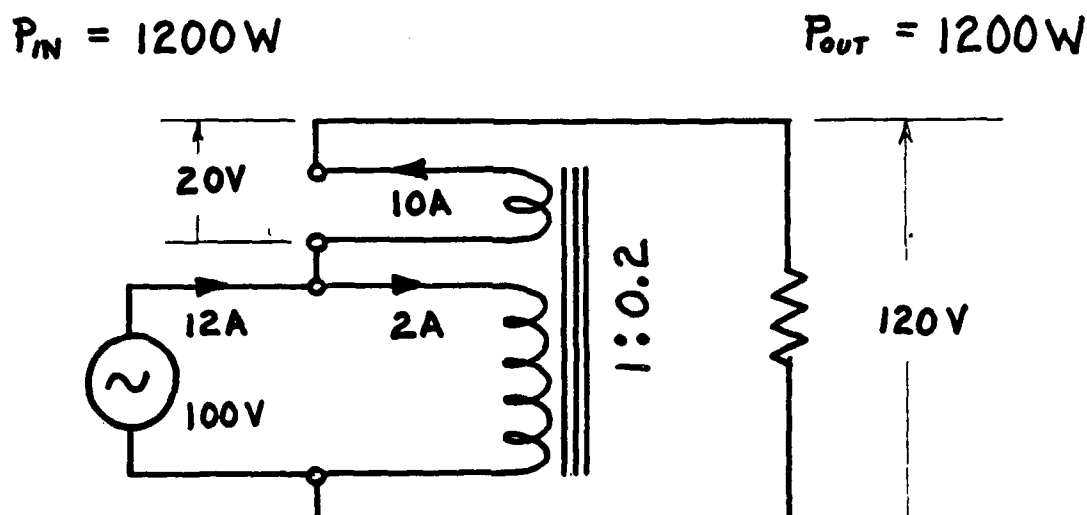


Figure 1B Frequency spectrum of waveform shown in Figure 1A



TRANSFORMER VOLT·AMP CAPACITY = 1200 VA

A. CONVENTIONAL TRANSFORMER



TRANSFORMER VOLT·AMP CAPACITY 200 VA

B. AUTOTRANSFORMER

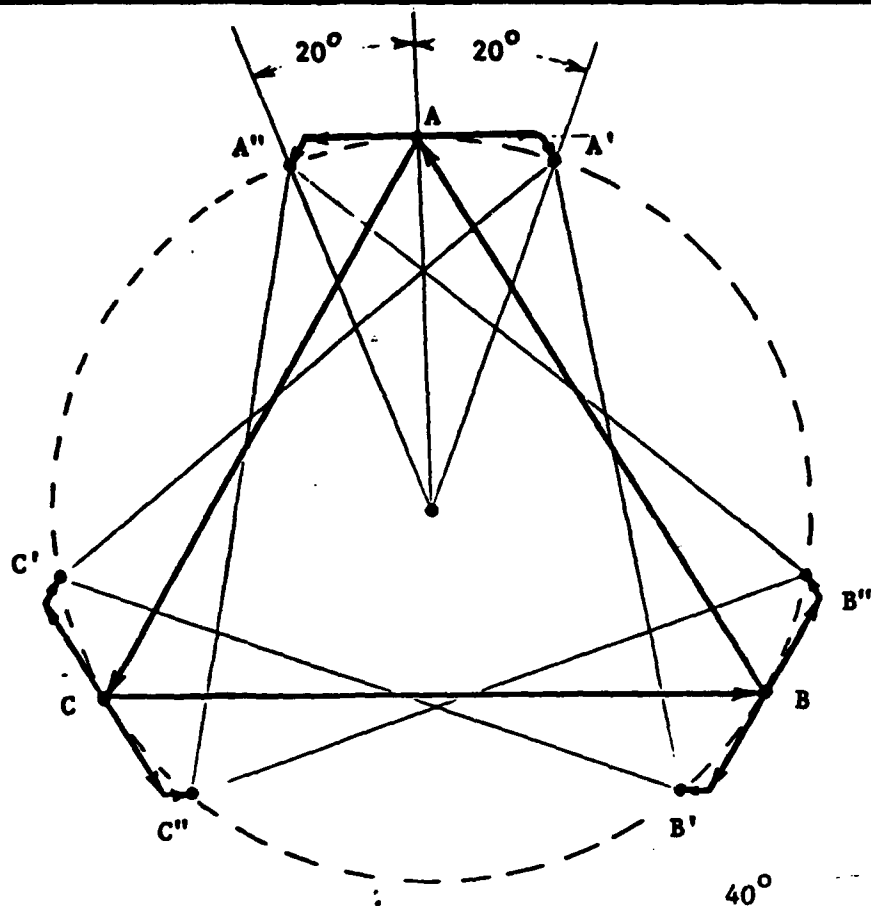
FIGURE 2 COMPARISON OF VOLT·AMP CAPACITY REQUIREMENTS OF CONVENTIONAL TRANSFORMERS AND AUTOTRANSFORMERS



The proposed method converts the 3-Phase input power to a 9-Phase system using autotransformer techniques as shown in Figures 3 and 4. In these figures it may be seen that <sup>output</sup> phases A, B, and C are provided directly by the input power lines, thereby requiring no transformation. Phases A'', A', B'', B', C'', C' are produced by transformation. However, as may be seen from Figure 3, these auxiliary phases are produced by adding relatively small phasors to the existing phases A, B, C. Only the short phasor paths such as exist between A and A', A and A'', etc. must be produced via transformer action. Since these phasor components are small relative to the phasors A-B, B-C, C-A from which they must be transformed, the effective transformation ratio between phase A' and phase A, etc. is close to unity. The volt-ampere capacity of the transformers required to generate phases A'', A', B'', B', C'', C' from phases A, B, C is therefore small compared with the volt-ampere capacity of conventional transformers that would provide complete DC isolation of the secondaries from the primaries. In off-line switch-mode power supplies, DC isolation is provided by a small high frequency transformer. Therefore, there is no need to provide DC isolation at the primary input power level.

Two versions of the 3-phase to 9-phase autotransformer conversion system are described. A 20 degree system is the most efficient. However, the use of this system requires that the load be split into three equal parts, with each part fed from an individual 3-phase bridge rectifier. If a 9-phase full-wave bridge rectifier were used with the 20 degree system, rectifier conduction would occur via paths wherein the line-to-line voltage exceeds the normal line-to-line voltage of the 3-phase power input, such as across phases A''-B', A'-C'', etc. shown in Figure 3-a.

A 40 degree system permits the use of a conventional 9-phase bridge rectifier to form a single rectified DC output. In this system, the auxiliary phases A'', A', B'', B', C'', and C' are arranged such that their combination with the input phases A, B, and C forms a uniformly distributed 9-phase system with 40 degrees between phases. The arrangement of phases for this system is shown in Figure 3-b.



a. 20 Degree System

b. 40 Degree System

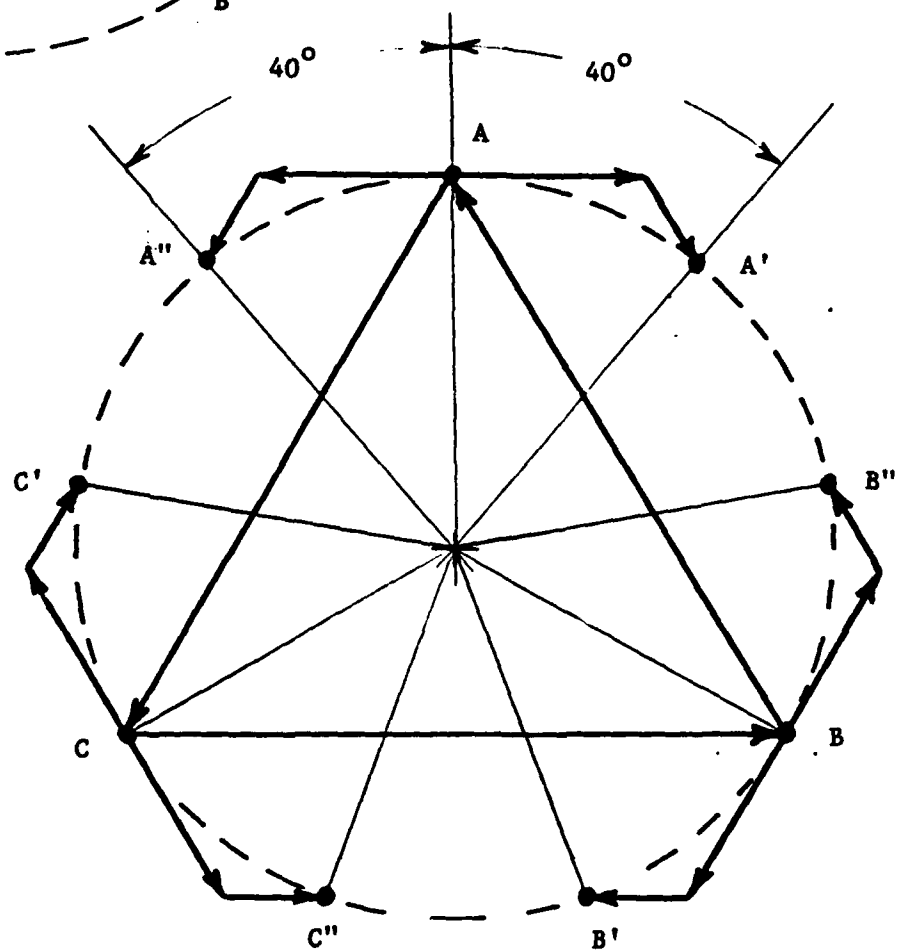


Figure 3 3-Phase to 9-Phase Conversion Using Autotransformer Method

As an example of the efficiency of the autotransformer technique, it is assumed that a 3-phase power source must supply 3000 Volt-Amperes at a line-to-line voltage of 115 volts to a group of switch-mode power supplies employing off-line rectifiers. It is also assumed that the power supplies may be grouped such that three equal 3-phase loads are formed. Assuming sinusoidal waveforms for simplicity, the line current in each of the three input power lines would be 15.1 amperes. A conventional isolating transformer system would be required to have a 3000 VA capacity in order to provide the required power. For the autotransformer method however, the current provided by phases A", A', B", B', C", C' would be one third of the 15.1 amperes or 5.03 amperes. *For the 20 degree system,* The short phasors which must be added to phases A,B,C to make these auxiliary phases are composed of two components each of length 20.40 volts and 4.62 volts, as shown in Appendix A. The volt-ampere capacity required to produce each of the auxiliary phases such as A" is

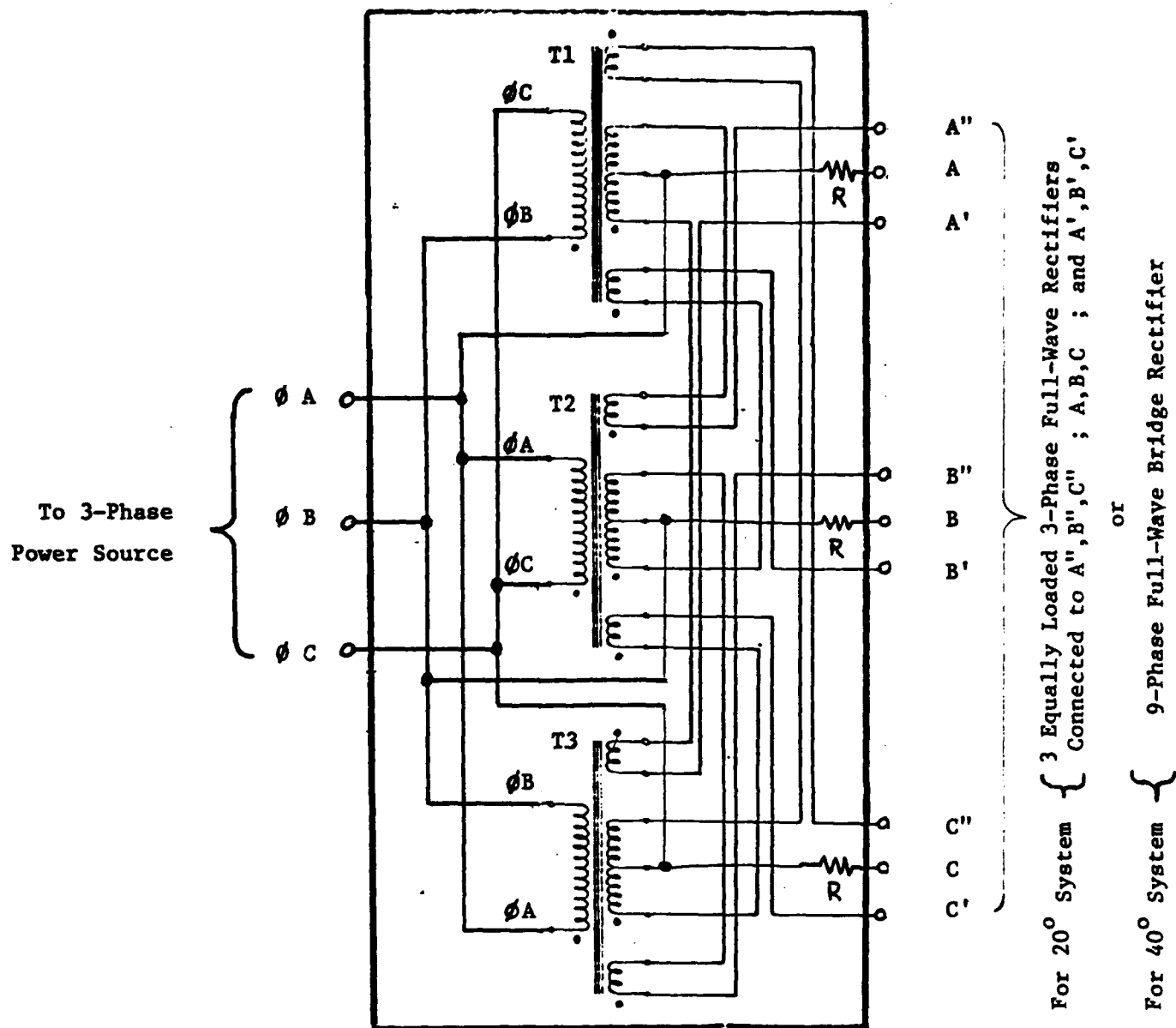
therefore  $(20.40V + 4.62V) \times 5.03A$  or 126 VA. For the six auxiliary phases, a total transformer capacity of 751.6 VA is required. *When the 20 degree system is used,* The autotransformer method therefore requires only 25 percent, approximately, of the volt-ampere capacity of conventional isolating transformers, resulting in a substantial reduction in size, weight, cost, acoustic noise, and stray magnetic fields.

Assuming approximately  $3.0 \text{ VA/in}^3$  as a reasonable sizing factor for  $60\text{Hz}$  transformers of this power range, a total of  $251 \text{ in}^3$  would be required to package the autotransformers. When related to an assumed input power level of 3000 watts, the resulting power density is  $12 \text{ Watts/in}^3$ . This level of power density would seem to make the autotransformer technique feasible for use in many applications. In those applications requiring a single rectified DC output, the 40 degree system must be used. The 40 degree system permits the use of a conventional 9-phase full-wave bridge rectifier. In this system however, the power density is reduced to approximately  $6 \text{ Watts/in}^3$ .

Since the reduction in line current harmonics may permit the use of a smaller EMI filter in many cases, the net impact upon system volume may be less than  $251 \text{ in}^3$ .

Figure 4 presents the schematic diagram of the 3-phase to 9-phase autotransformer conversion system. Transformers connected in this manner will convert a balanced 3-phase input to the 9-phase system shown in Figure 3. Figure 5A indicates the waveform of the current flowing in the 3-phase input power lines when either a 20 degree or 40 degree 3-phase to 9-phase autotransformer conversion system is used. The individual rectifier current pulses are assumed to be trapezoidal, 10 degrees wide at the top and 30 degrees wide at the base. This pulse waveform is typical of that occurring in capacitor input off-line rectifiers. The line current waveform shown in Figure 5A is nearly sinusoidal, having a total harmonic distortion (THD) of only 5 percent. The frequency spectrum of this waveform is shown in Figure 5B. The residual distortion is predominantly 17th and 19th harmonic, having amplitudes of 3.9 percent and 3.0 percent respectively. This waveform is dramatically improved over that of the conventional 3-phase off-line rectifier line current shown in Figure 1A.

Because the output phases A,B,C are provided directly by the input phases  $\phi A, \phi B, \phi C$ , their output impedances will be lower than the other phases which are produced by the transformers. In order to provide equal source impedances for each of the output phases, a resistance R is placed in series with the A,B,C output phases as shown in Figure 4. The value of these resistors should be chosen to equal the effective series resistance inserted in the auxiliary phase outputs A'',A',B'',B',C'',C' by the autotransformers. The value of this resistance may be measured between the  $\phi A$  input and the A'' output, with  $\phi A, \phi B$ , and  $\phi C$  shorted together. Improved matching of the output impedances would occur if the resistors were inductively wound so as to match the inductive component of the output impedances as well.



Basic Transformer Specifications for 115 V Line-to-Line 3000 VA System			
	20 Degree System	40 Degree System	6.67 Degree System
Primary	115 V	115 V	115 V
Secondary #1 and #3	4.624 V	17.936 V	Not Used
Secondary #2	40.794 V CT	67.418 V CT	15.521 V CT
Phase Shift	0 Degrees	0 Degrees	0 Degrees
VA Rating per Trans	250 VA	520 VA	78 VA

Figure 4 3-Phase to 9-Phase Autotransformer Conversion System

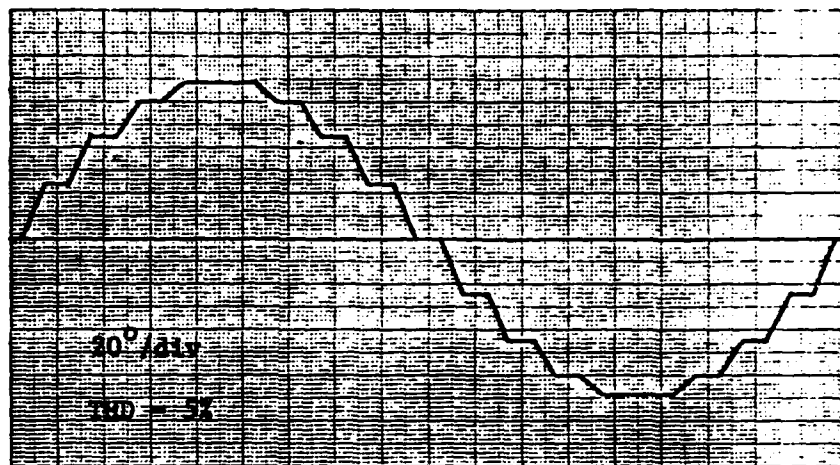


Figure 5A Waveform of current flowing in the phase A,B, and C input power lines with a 20° or 40° 3-phase to 9-phase autotransformer conversion system feeding off-line rectifiers

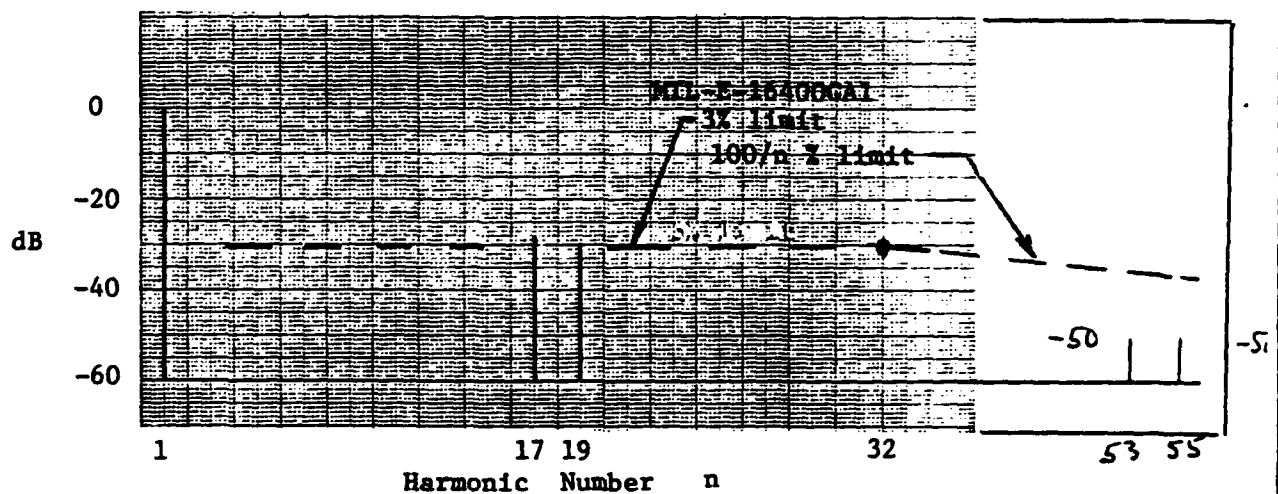


Figure 5B Frequency spectrum of waveform shown in Figure 5A

The improvement is achieved primarily by converting the waveform shown in Figure 1-a into the sum of three similar waveforms of one third amplitude, with one of the waveforms advanced by 20 degrees (or 40 degrees, depending upon the system used) and one retarded by 20 degrees ( or 40 degrees) relative to the third waveform.

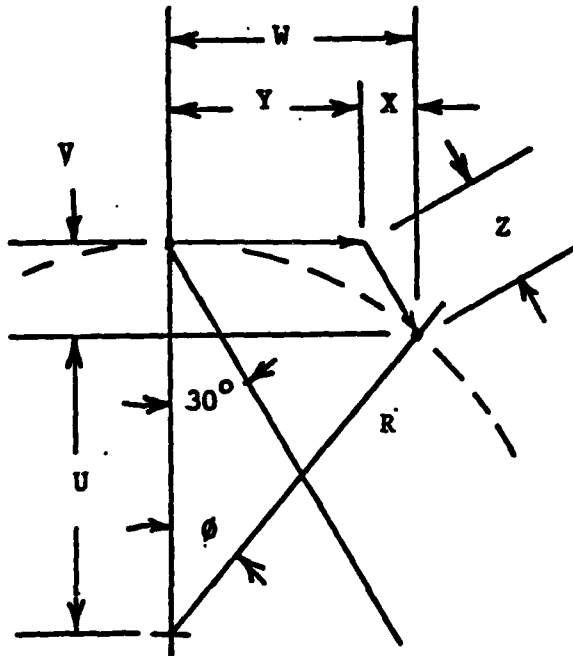
The summation of these three waveforms is then substantially more similar to a sine wave than is the severe pulse waveform shown in Figure 1A. The remaining improvement results from the fact that the transformer primary currents, which are reduced amplitude replicas of their secondary currents, very conveniently add to and subtract from the waveform in an optimum manner to make the total waveform very nearly sinusoidal. This process is discussed in detail in Appendix B.

The analysis presented herein assumes perfectly balanced 3-phase voltages. Imbalance of the input phase voltages would cause some degradation of performance. A quantitative analysis of the effects of phase voltage imbalance has yet to be performed.

Approximately 23 dB further reduction of the 17th and 19th harmonic could be achieved in large systems by cascading 3-phase to 9-phase autotransformer conversion systems, as discussed in Appendix C.

# APPENDIX A

## DETERMINATION OF TRANSFORMER SECONDARY VOLTAGE RATINGS FOR TRANSFORMERS USED IN A 3-PHASE TO 9-PHASE AUTOTRANSFORMER CONVERSION SYSTEM



R = Radius

$$U = R \cos \phi$$

$$V = R - U$$

$$W = R \sin \phi$$

$$X = V \tan 30^\circ$$

$$Y = W - X$$

$$Z = V / \cos 30^\circ$$

$$\begin{aligned} \text{Radius} &= \frac{V_{LL}}{2 \cos 30^\circ} \\ &= 66.395 \text{ V} \\ &\text{@ } V_{LL} = 115 \text{ V} \end{aligned}$$

From inspection of the figure above, it may be seen that the transformer secondary voltages required for the 3-phase to 9-phase autotransformer conversion system are given by the following equations.

$$\begin{aligned} V_Y &= \frac{V_{LL}}{2 \cos 30^\circ} \left( \sin \phi - (1 - \cos \phi) \tan 30^\circ \right) \\ V_Z &= \frac{V_{LL}}{2 \cos 30^\circ} \left( \frac{1 - \cos \phi}{\cos 30^\circ} \right) \end{aligned}$$

	20 Degree System	40 Degree System	6.67 Degree System
Voltage with $V_{LL} = 115 \text{ V}$			
$V_Y$	20.3968 V	33.709 V	7.7605 V
$V_Z$	4.6236 V	17.936 V	Not Used
Turns Y : Primary	0.1774 : 1	0.2931 : 1	0.06748 : 1
Ratios Z : Primary	0.0402 : 1	0.1560 : 1	Not Used



## APPENDIX B

### PROCEDURE FOR DETERMINING THE WAVEFORM OF THE INPUT LINE CURRENT FOR A 3-PHASE TO 9-PHASE AUTOTRANSFORMER CONVERSION SYSTEM FEEDING OFF-LINE RECTIFIERS

The waveform of the input line current is determined by the use of a procedure outlined below in which the instantaneous current in each of the bridge rectifier paths is determined over a full cycle of the input power. The current flowing in the input power lines is then determined by summation of appropriate rectifier currents and transformer primary currents with the transformer primary current determined by reflecting the secondary currents thru the turns ratios which have been established in Appendix A.

For the 20 degree system, it is assumed that three identical 3-phase rectifiers having equal loads are connected respectively to terminals A", B", C" ; A,B,C; and A',B',C'. It should be noted that the use of a single 9-phase bridge rectifier is not permissible with the 20 degree system, because rectifier conduction would occur via sneak paths such as A"-B'. The sneak path line-to-line voltages exceed the normal line-to-line voltages, such as A-B. For the 40 degree system, it is assumed that a 9-phase full-wave bridge rectifier is connected to terminals A",A,A',B",B,B',C",C,C'.

It is also assumed that the rectifier conduction is in the form of a symmetrical trapezoid 20 degrees wide at the 50 percent points, 30 degrees wide at the base, and 10 degrees wide at the top. For simplicity, it is assumed that the rectifier current pulse occurs at such a time that the center of the current pulse coincides with the time at which the line-to-line voltage which causes the conduction is at a peak.

The timing of the center of the rectifier conduction pulses is determined using the phasor diagram of Figure B1. A counter-clockwise rotation of the phasor diagram is assumed. Peak line-to-line voltage is assumed to occur when the line-to-line voltage phasor is vertical.

a. 20 Degree System

For  $V_{LL} = 115 \text{ V rms}$

Phasors A-B, A''-B'', etc  
 = 115 V rms  
 = 162.63 V peak

b. 40 Degree System

For  $V_{LL} = 115 \text{ V}$

Phasors A-B', C-A', etc  
 = 130.77 V rms  
 = 184.94 V peak

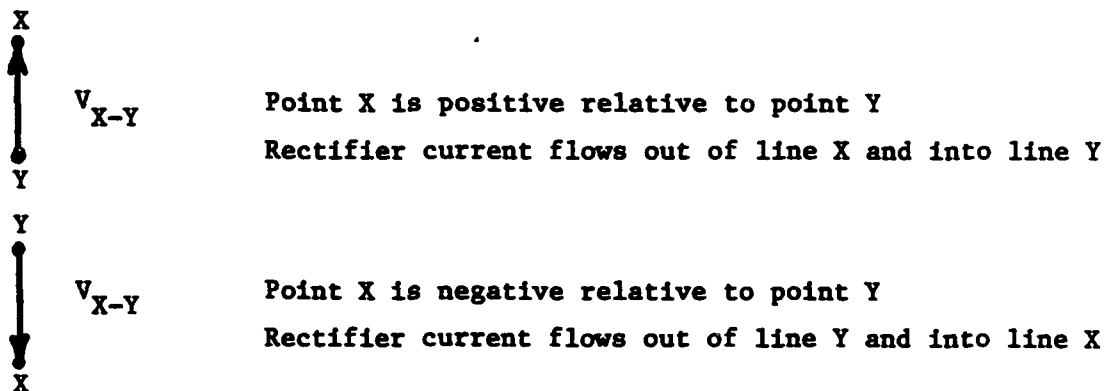
Figure B1 Line-to-Line Voltage Phasor Diagram for 3-Phase to 9-Phase Autotransformer Conversion System

Because the use of full-wave bridge rectifiers is assumed, conduction will occur for both peak positive and peak negative line-to-line voltages. The direction of line current flow is determined by whether the phasor is pointing upward or downward.

Because the process of constructing the entire waveform for the input line current is lengthy and tedious, a formal procedure has been established. This permits an orderly process, and documents the rationale. In order to greatly simplify the process, it is first assumed that the diode conduction occurs as current impulses of zero width. The final waveform is then formed by converting the impulses into trapezoidal pulses having the same amplitude. The procedure for constructing the waveform is given below.

1. The phasor diagram of Figure B1 is rotated CCW.

The rectifier bridge is assumed to conduct when the relevant line-to-line source voltage phasor is vertical. The direction of line current flow is defined below.



2. Current that flows out of line A' or A" also flows out of line  $\emptyset A$   
 Current that flows out of line B' or B" also flows out of line  $\emptyset B$   
 Current that flows out of line C' or C" also flows out of line  $\emptyset C$

3.	Current that flows out of line:	20°	40°
	A' [B'] (C')	System	System
	also flows out of line ØB [ØC] (ØA) reduced by:	0.1774	0.2931
	also flows into line ØC [ØA] (ØB) reduced by:	0.1774	0.2931
	also flows out of line ØB [ØC] (ØA) reduced by:	0.0402	0.1560
	also flows into line ØA [ØB] (ØC) reduced by:	0.0402	0.1560
	Current that flows out of line:	20°	40°
	A'' [B''] (C'')	System	System
	also flows out of line ØC [ØA] (ØB) reduced by:	0.1774	0.2931
	also flows into line ØB [ØC] (ØA) reduced by:	0.1774	0.2931
	also flows out of line ØC [ØA] (ØB) reduced by:	0.0402	0.1560
	also flows into line ØA [ØB] (ØC) reduced by:	0.0402	0.1560

Items 2 and 3 above may be more easily understood by examining the transformer system schematic diagram, Figure B4.

The rules given in item 2 should be obvious from inspection of Figure B4. In considering the rules of item 3, it is helpful to remember that current flowing out of a dotted transformer secondary terminal must flow into the dotted primary terminal, reduced by the appropriate turns ratio. It should also be noted that current flowing into a transformer primary terminal must flow out of the primary power line connected to that transformer terminal.

Figure B2 presents the impulse representation of the rectifier currents flowing out of the respective transformer system output terminals, for a 20 degree system.

Figure B2 is formed by examination of Figure B1 and the application of rule 1 above. By examination of Figure B1, it may be seen that the phasor C''-A'' is vertically downward after 10 degrees of rotation. Accordingly, a *positive* current impulse is shown for terminal A'' and a *negative* current impulse is shown for terminal C'' at an angle of 10 degrees. The polarities of these impulses are determined in accordance with rule 1. At a rotation of 30 degrees, phasor C-A is positioned vertically downward, resulting in appropriate current impulses shown for terminals A and C. This process is continued for a full 360 degree rotation of the phasor diagram.

Rule 2 is then applied to form the intermediate impulse waveform labeled Ax1.000 in Figure B3. This waveform is simply the direct summation of the impulse waveforms A'', A, and A' shown in Figure B2. The rules of item 3 are then applied to form the intermediate impulse waveforms labeled Ax0.1774 and Ax0.0402 shown in Figure B3. These two waveforms account for the line current flowing out of the input power line A as a result of the current pulses flowing in the primaries of the transformers connected to line A.

The impulse waveforms labeled Ax1.0000, Ax0.1774, and Ax0.0402 are then summed, accounting for the scale factors to form the impulse waveform labeled Asum, which represents in impulse form the total current flowing out of the phase A input power line. This impulse waveform is then converted to the actual time waveform of the line current flowing out of the phase A input power line replacing the impulses of the Asum waveform with trapezoidal pulses. The actual time waveform is then equal to the instantaneous sum of the trapezoidal pulses. A similar process will yield the waveform of the line current flowing in the phase B and phase C input power lines. By the symmetry of the system, it may be seen that these waveforms are identical to that of the phase A waveform, except for a plus or minus 120 degree phase shift of the fundamental.

Figure B4 illustrates the flow of rectifier current and the resulting transformer primary currents flowing at 10 degrees rotation of the phasor diagram, Figure B1. In Figure B4, the bridge rectifier current flows out of terminal A'' and into terminal C''. All other rectifier terminals have zero conduction at this time, since the associated diodes are reverse biased.

Figure B5 presents the impulse representation of the rectifier currents for a 40 degree system. Figure B6 illustrates the formation of the actual time waveform for the phase A input line current for a 40 degree system.

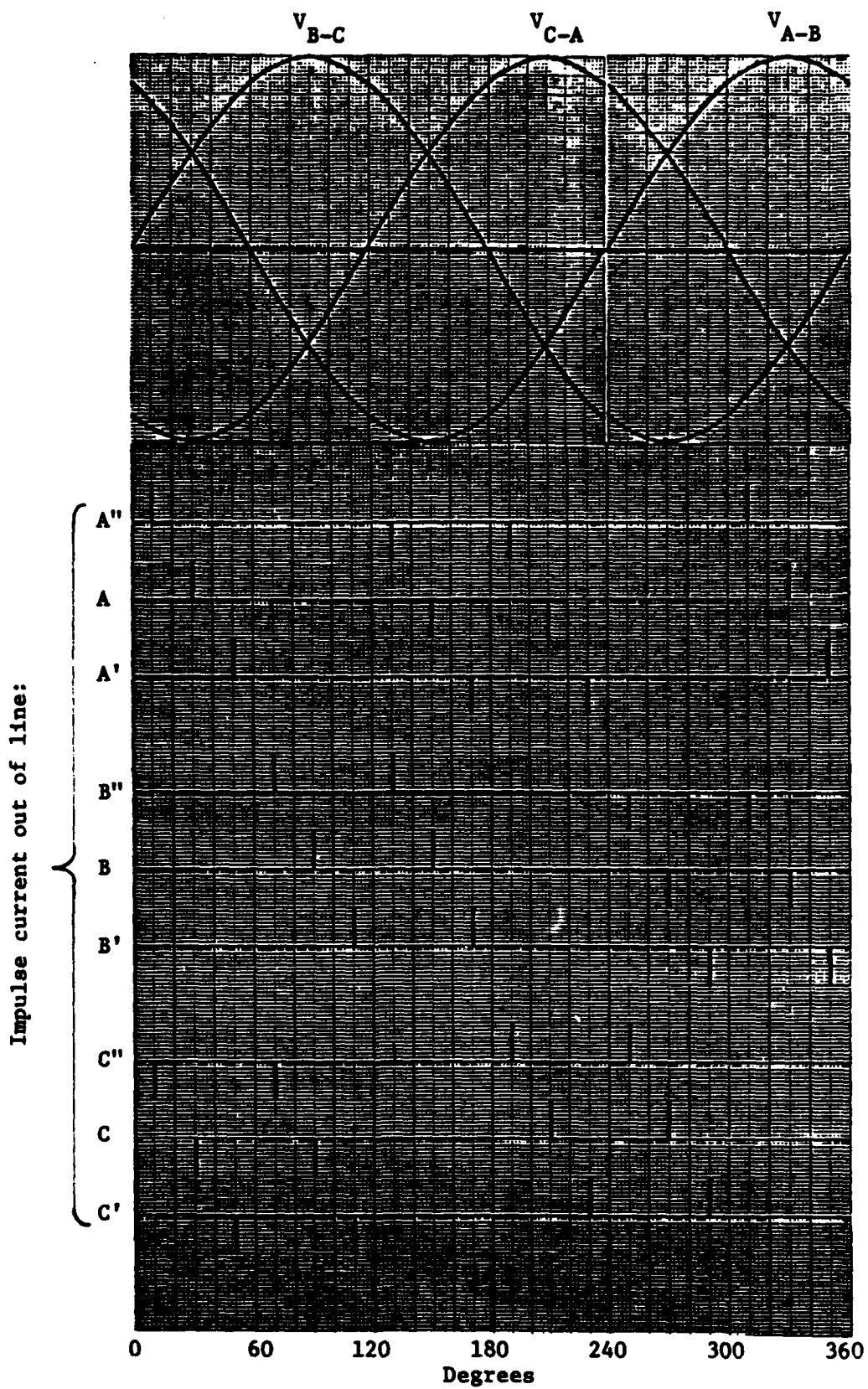


Figure B2 Timing of Rectifier Current Pulses for 20 Degree System

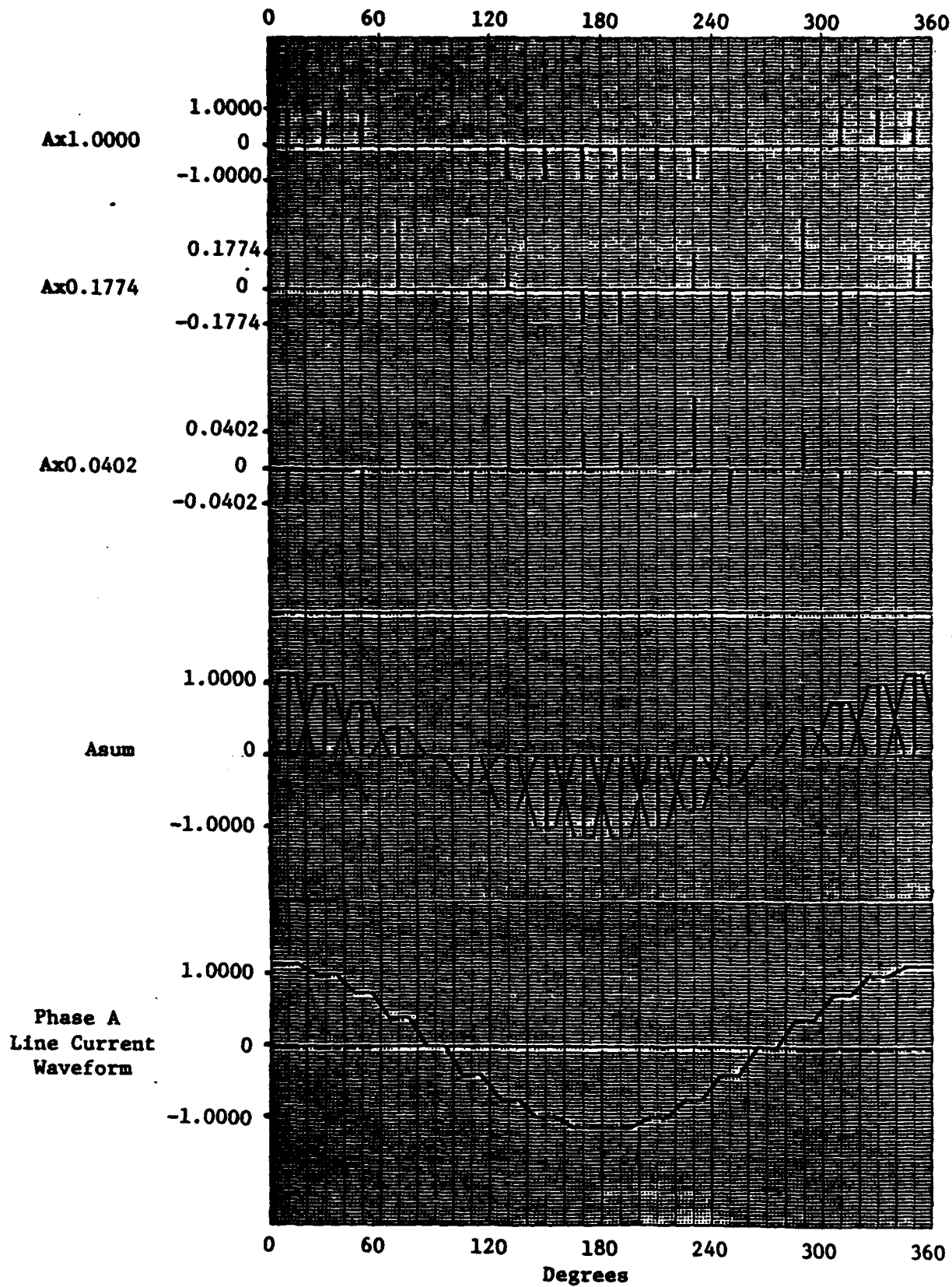


Figure B3 Formation of Phase A Line Current Waveform by Summation of Component Impulses and Convolution with Rectifier Current Pulseform for 20 Degree System

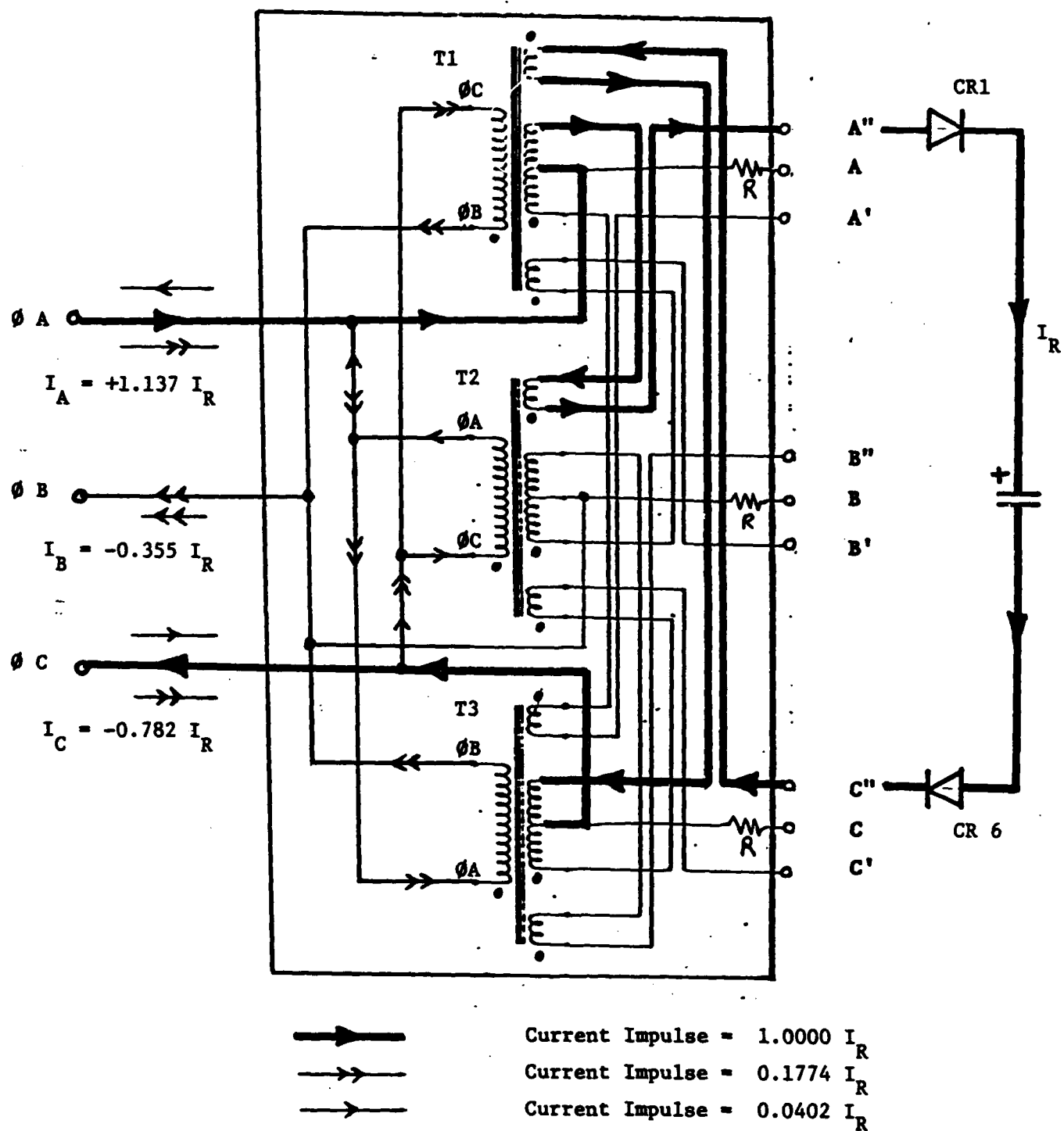


Figure B4

Trace of Rectifier Impulse Current Flow for 20 Degree System  
at 10 Degrees Rotation of Figure B1-a Phasor Diagram



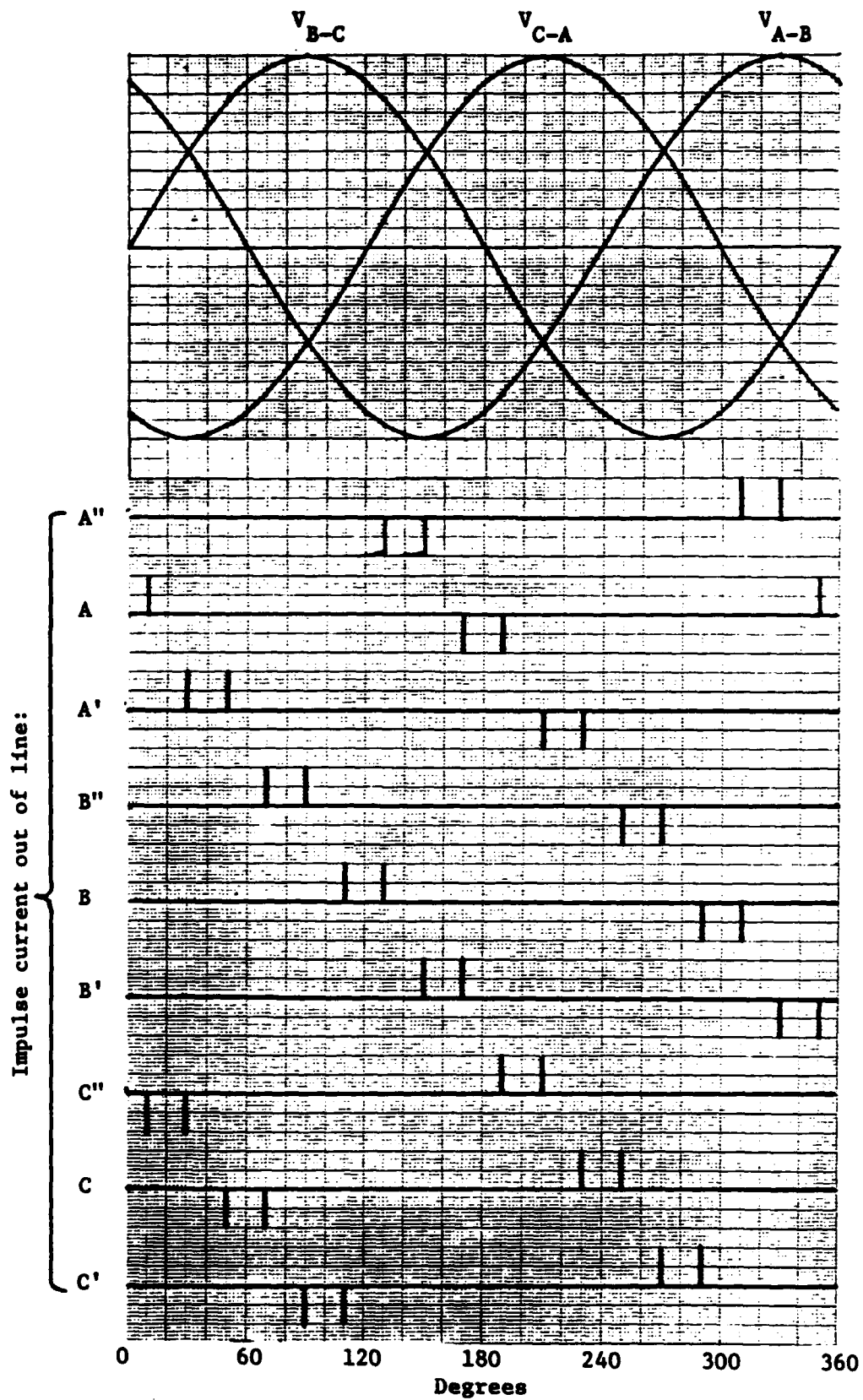


Figure B5 Timing of Rectifier Current Pulses for 40 Degree System

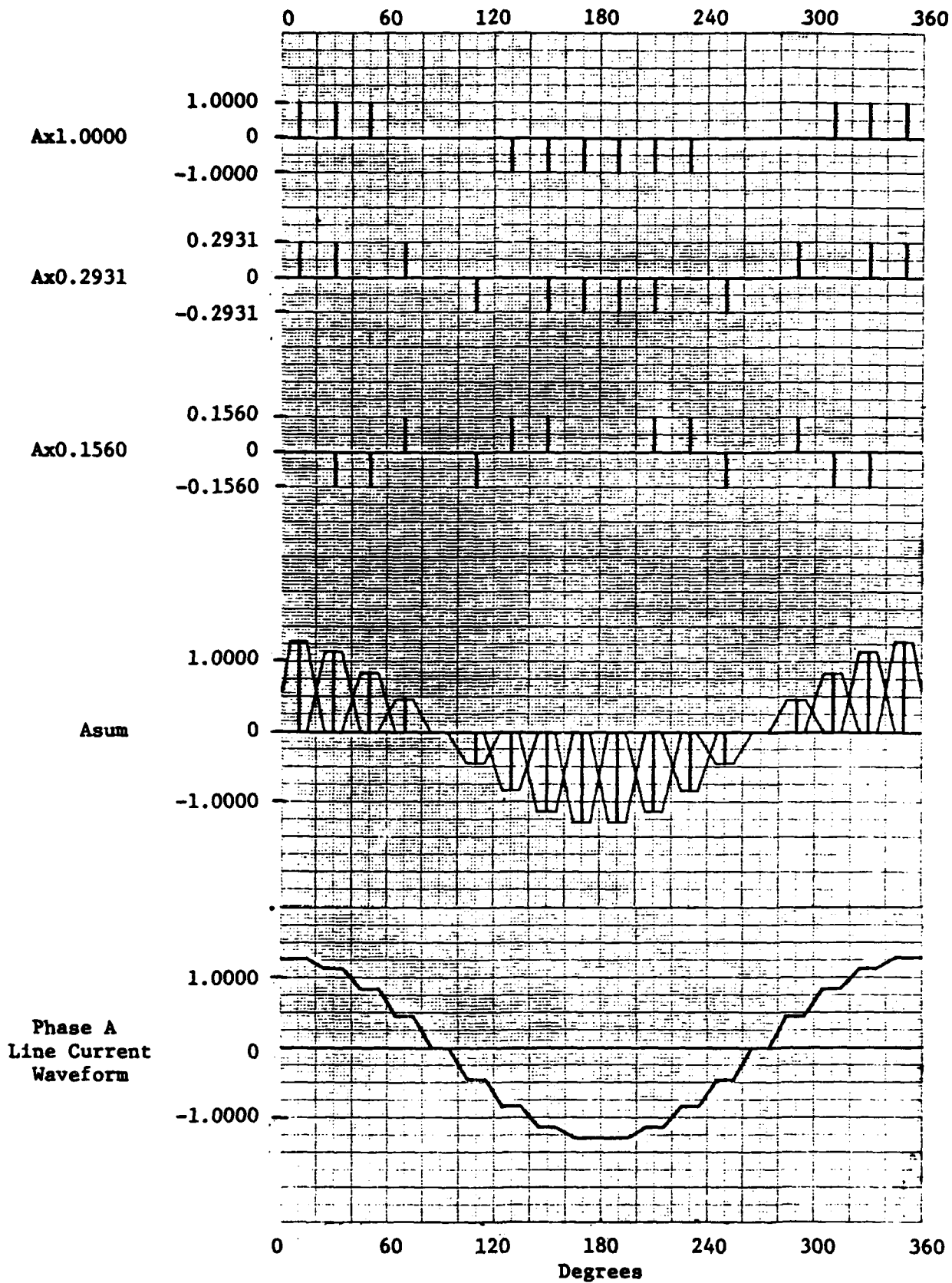


Figure B6

Formation of Phase Line Current Waveform by Summation of Component Impulses of Rectifier Current for 40 Degree System

## APPENDIX C

### REDUCTION OF 17th AND 19th HARMONIC LINE CURRENTS BY CASCADING 3-PHASE TO 9-PHASE AUTOTRANSFORMER CONVERSION SYSTEMS

The 3-phase to 9-phase autotransformer conversion system described in the body of the report provides a nearly sinusoidal line current waveform. There is approximately 5 percent residual distortion, however, which is predominantly 17th and 19th harmonic. These harmonics have amplitudes of 3.9 percent and 3.0 percent respectively. In large systems that have a number of power supplies which can be allocated so as to form three approximately equal loads, approximately 23 dB of attenuation of the 17th and 19th harmonic currents can be effected by the use of cascaded autotransformer conversion systems. Figure C1 presents a block diagram of such an arrangement.

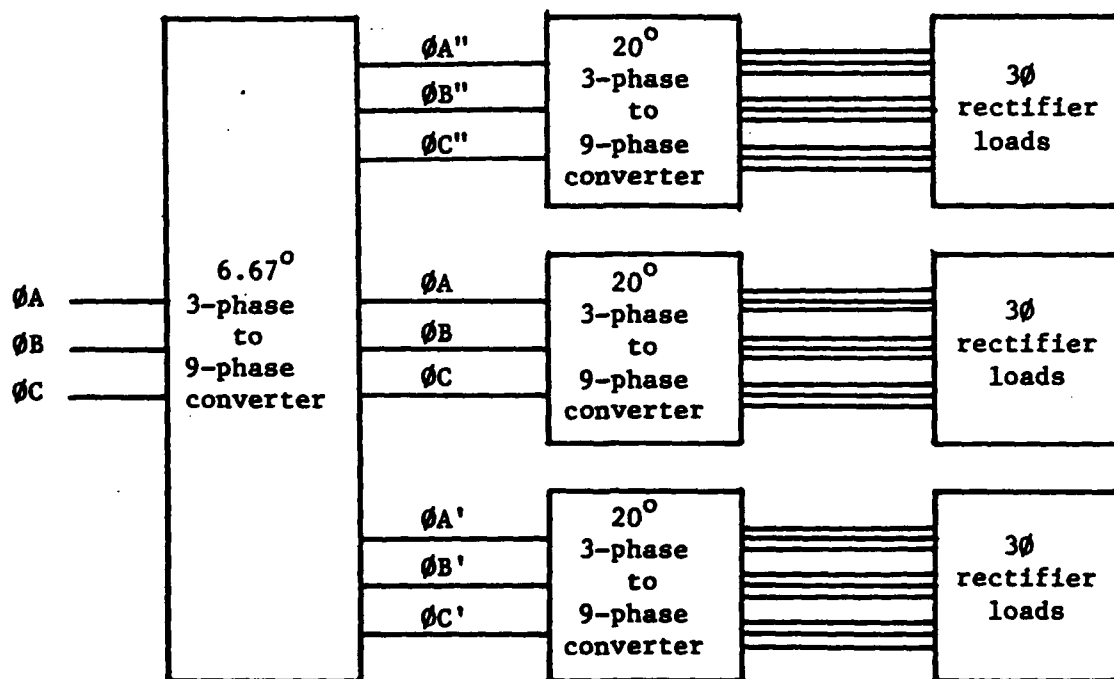


Figure C1 Cascading of 3-phase to 9-phase Autotransformer Conversion Systems to Provide 23 dB Reduction of 17th and 19th Harmonic Line Current

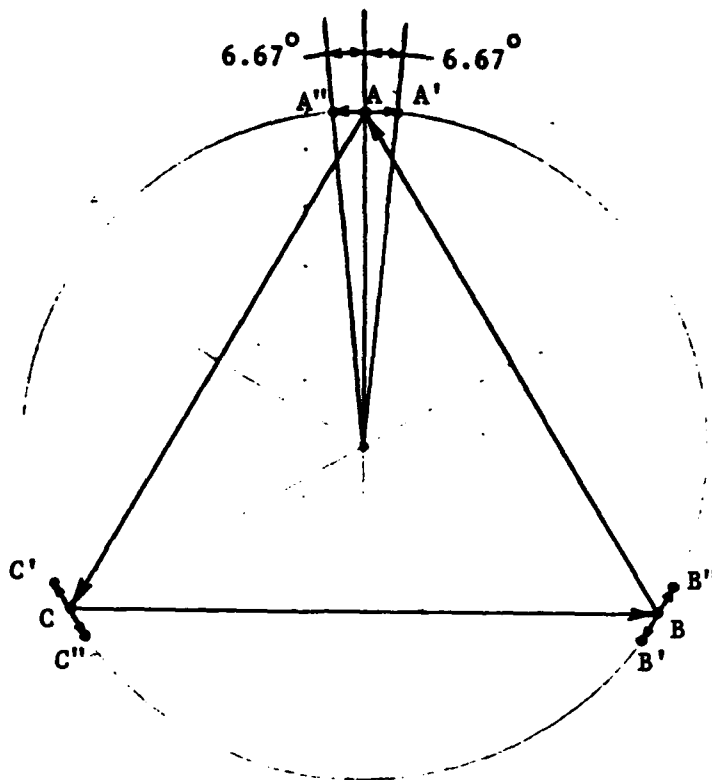


Figure C2 Phasor Diagram of 6.67° 3-Phase to 9-Phase Autotransformer Conversion System

The phasor diagram for the 6.67 degree 3-phase to 9-phase autotransformer conversion system is shown in Figure C2. Because of the very small angular separation of the points A'', A, and A', the auxiliary phases A'', A', B'', B', C'' and C' may be produced by the addition of single small phasors to the input phases A, B, and C. Also because of the very small angle, the transformed voltages such as A''-A are very small compared with the phases from which they must be transformed, such as B-C. The volt-ampere capacity of the transformers required to provide the small phasors may therefore be very much smaller than the volt-ampere capacity of the system which is fed from the autotransformer conversion system. For example, assuming a 30 KVA system, with 115 volts line-to-line, the input line current from the primary power source would be 150 amperes. With the system shown, 50 amperes would be provided by each of the auxiliary phases A'', A', B'', B', C'', C'. The small phasor voltages provided by the transformers are equal to 7.761 volts. A total volt-ampere requirement for the transformers is therefore equal to 7.761 Volts x 50 Amperes x 6 = 2328 VA for the assumed 30 KVA system. Only

7.8 percent of the system volt-ampere capacity must be provided by transformer action of the 6.67 degree 3-phase to 9-phase autotransformer conversion system.

The 6.67 degree shift of the fundamental would provide a plus and minus 120 degree shift of the 18th harmonic. When combined with the unshifted component, complete cancellation of the 18th harmonic would occur, except for the fact that the 18th harmonic is nonexistent. For the 17th harmonic, a shift of plus or minus 113.33 degrees would occur. Cancellation would therefore be incomplete, but would amount to a value of  $(1 + 2\cos 113.33^\circ) / 3.0 = 0.0693 = 23.2 \text{ dB}$ . For the 19th harmonic the phase shift is plus or minus 126.67 degrees. Similarly, the attenuation of the 19th harmonic is 23.8 dB. A detailed analysis of the system shown in this appendix has yet to be performed. However, it is expected that a design of this nature could achieve substantial values of attenuation of the 17th and 19th harmonic currents which are the predominant components of the 5 percent residual distortion of the line current at the input of a 20 degree 3-phase to 9-phase autotransformer conversion system. Using a  $3 \text{ VA/in}^3$  sizing factor for transformers, the power density of the 6.67 degree converter would be approximately  $39 \text{ Watts/in}^3$ . This order of power density would make such an approach attractive in many applications.

# APPENDIX D

## COMPARISON OF RELATIVE AMPLITUDES AND WAVESHAPES OF INPUT LINE CURRENT WITH AND WITHOUT THE 3-PHASE TO 9-PHASE AUTOTRANSFORMER CONVERSION SYSTEM

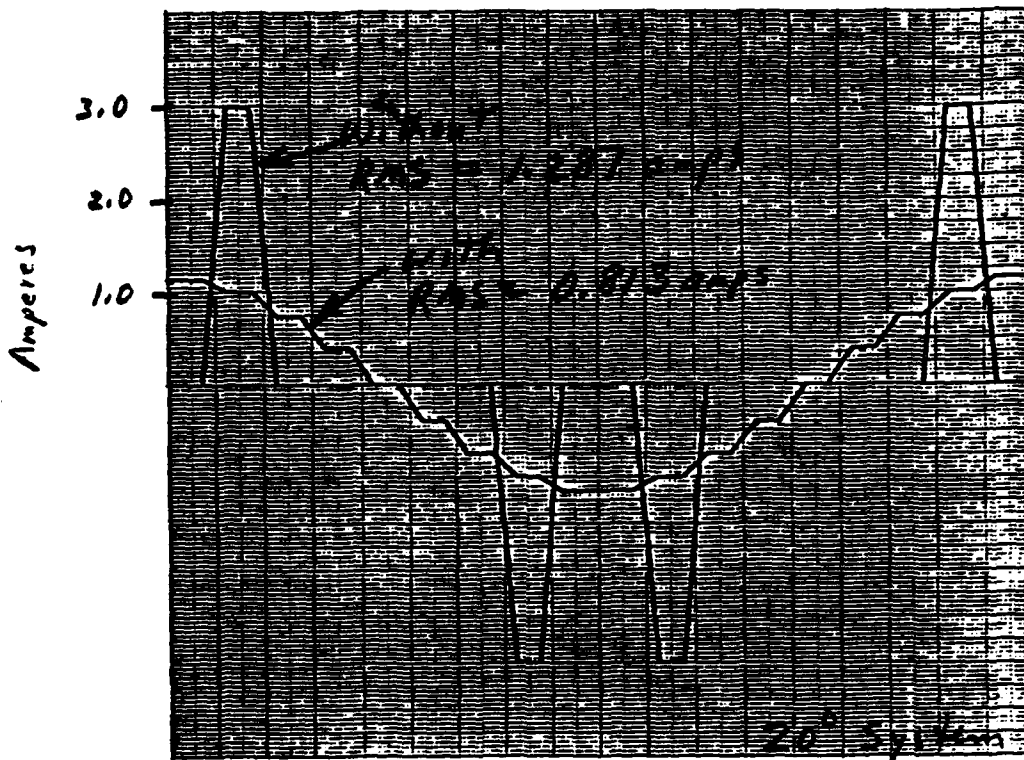


Figure D1 Relative Waveforms of Current Flowing in Input Power Lines With and Without 3-Phase to 9-Phase Autotransformer Conversion System. Relative Magnitudes are Drawn to Scale. for 1.0 amps DC Rectified output Current

$$P_{OUT} = (162 \text{ VDC})(1.0 \text{ ADC}) = 162 \text{ Watts}$$

$$\text{Volt} \cdot \text{Amp}_{IN \text{ WITH}} = (66.4 \text{ Vrms})(0.813 \text{ Arms})(3) = 162 \text{ VA}$$

$$\text{Power Factor} = 1.0$$

$$\text{Volt} \cdot \text{Amp}_{IN \text{ WITHOUT}} = (66.4 \text{ Vrms})(1.287 \text{ Arms})(3) = 256 \text{ VA}$$

$$\text{Power Factor} = 0.63$$

## APPENDIX D

### COMPARISON OF RELATIVE AMPLITUDES AND WAVESHAPES OF INPUT LINE CURRENT WITH AND WITHOUT THE 3-PHASE TO 9-PHASE AUTOTRANSFORMER CONVERSION SYSTEM

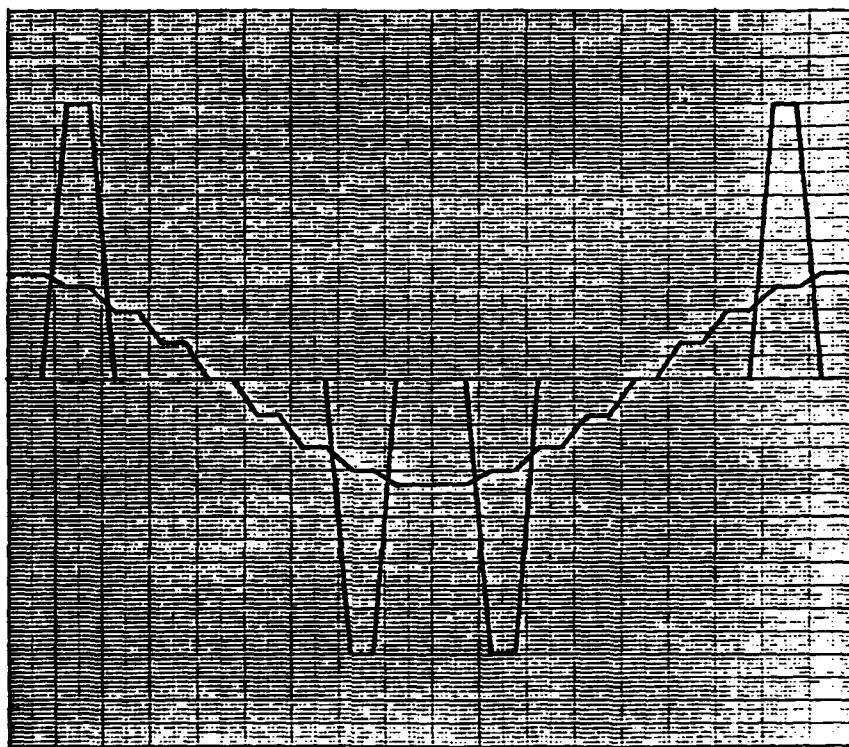


Figure D1      Relative Waveforms of Current Flowing in Input Power Lines With and Without 3-Phase to 9-Phase Autotransformer Conversion System. Relative Magnitudes are Drawn to Scale.

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